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# Reactions of Aliphatic Diazo Compounds: III.\* Reaction of Ethyl Diazoacetate with 1,3-Diarylpropenones\*\*

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**Abstract**—Ethyl diazoacetate adds to 1,3-diarylpropenones in a regioselective fashion to give intermediate 4,5-dihydro-3*H*-pyrazole derivative; 1,3-hydride shift in the latter leads to formation of isomeric ethyl 4-aryl-5-aroyl-4,5-dihydro-1*H*-pyrazole-3-carboxylate and ethyl 4-aryl-3-aroyl-4,5-dihydro-1*H*-pyrazole-5-carboxylate at a ratio of 5:1. Thermolysis of these products is not stereospecific; as a result, three isomeric substituted ethyl cyclopropanecarboxylates and 2-pyranone derivative are formed.

Although 1,3-dipolar cycloaddition reactions of diazo compounds with unsaturated compounds possessing electron-acceptor groups were known for a long time, in some cases the structure of the adducts was not established unambiguously [2]. For example, Kohler and Steele [3] showed that the reaction of ethyl diazoacetate (I) with 1,3-diphenylpropenone (IIa) yields ethyl 5-benzoyl-4-phenyl-4,5-dihydro-1*H*-pyrazole-3-carboxylate, whereas in the more recent study [4] the same product was assigned the structure

of ethyl 3-benzoyl-4-phenyl-4,5-dihydro-1*H*-pyrazole-5-carboxylate.

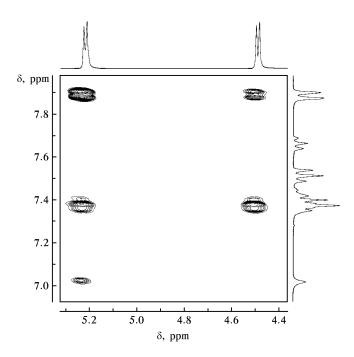
We have examined the reactions of ester I with 1,3-diarylpropenones IIa–IIi. By heating of a mixture of ethyl diazoacetate (I) with 1,3-diphenylpropenone (IIa) in heptane for 7 h at 75°C we obtained isomeric dihydropyrazoles IIIa and IVa which were isolated in 36 and 8% yield, respectively (Scheme 1). The structure of the products was determined on the basis of their elemental compositions (Table 1) and spectral

## Scheme 1.

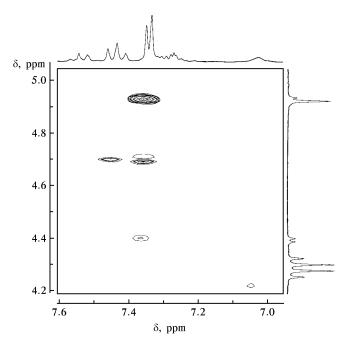
II-IV, R = R' = H (a); R' = H, R = 4-Me (b), 4-MeO (c), 3-Br (d), 4-Br (e), 2-Cl (f); R = H, R' = 4-MeO (g); R = 4-MeO, R' = 4-Cl (h); R = 3-Br, R' = 4-Cl (i).

<sup>\*</sup> For communication II, see [1].

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**Fig. 1.** <sup>1</sup>H–<sup>1</sup>H NOESY spectrum of ethyl 5-benzoyl-4-phenyl-4,5-dihydro-1*H*-pyrazole-3-carboxylate (**IIIa**).



**Fig. 2.** <sup>1</sup>H–<sup>1</sup>H NOESY spectrum of ethyl 3-benzoyl-4-phenyl-4,5-dihydro-1*H*-pyrazole-5-carboxylate (**IVa**).

data (Table 2). In the IR spectra of **IIIa** and **IVa** we observed absorption bands typical of NH (3400 cm<sup>-1</sup>) and C=O groups (1700 and 1740 cm<sup>-1</sup>). The absorption maxima in the UV spectrum of **IVa** are displaced to the long-wave region as compared to those

observed for compound **IIIa**:  $\lambda_{\text{max}}$  260 and 326 nm for **IVa** and 248 and 290 nm for **IIIa**. This indicates a more extended conjugation chain in molecule **IVa**. The <sup>1</sup>H NMR spectrum of pyrazole **IIIa** contains two doublets at  $\delta$  4.49 and 5.22 ppm (J = 4 Hz). In the spectrum of **IVa** the pyrazole ring protons give two doublets at  $\delta$  4.40 and 4.95 ppm (J = 4 Hz); also, signals from the ethyl group, aromatic rings, and NH group are present. The <sup>13</sup>C NMR spectra of **IIIa** and **IVa** are consistent with the assumed structures.

In order to prove the structure of products **IIIa** and **IVa** more rigorously, diphenylpropenone **IIa** was brought into reaction with ethyl diazoacetate-*d* [5]. The reaction afforded two products, one of which had physical properties and spectral parameters similar to those of **IIIa**. The other product showed in the <sup>1</sup>H NMR spectrum a singlet from 4-H of the pyrazole ring, indicating the presence of deuterium on the neighboring carbon atom to which the ester group is attached. This means that the second product has structure **VI**:

IIa 
$$\xrightarrow{N_2\text{CDCOOEt}}$$
 IIIa +  $\xrightarrow{D}$   $\xrightarrow{N}$   $\xrightarrow{N}$ 

The <sup>1</sup>H-<sup>1</sup>H NOESY spectrum of isomer **IIIa** (Fig. 1) displayed a cross peak due to coupling of the NH proton with C**H**COPh. In the <sup>1</sup>H-<sup>1</sup>H NOESY spectrum of isomer **IVa** (Fig. 2) we observed cross peaks corresponding to coupling between C**H**CO<sub>2</sub>Et and NH protons and between the former and *ortho*-protons of the phenyl (but not benzoyl) group. These data allowed us to rule out alternative structures of ethyl 4-benzoyl-5-phenyl-4,5-dihydro-1*H*-pyrazole-3-carboxylate and ethyl 4-benzoyl-3-phenyl-4,5-dihydro-1*H*-pyrazole-5-carboxylate.

The oxidation of compounds **IIIa** and **IVa** with bromine gave ethyl 3-benzoyl-4-phenylpyrazole-5-carboxylate (**VII**) whose structure was established on the basis of its spectral parameters and published data [4]:

Comp.	Yield, %	9C		Found, %		Formula	Calculated, %				
	i ieid, %	mp, °C	С	Н	N	Formula	С	Н	N		
IIIa	36	142	70.66	5.76	8.66	$C_{19}H_{18}N_2O_3$	70.80	5.59	8.69		
IIIb	47	147	71.32	6.02	8.33	$C_{20}^{10}H_{20}^{20}N_{2}^{2}O_{3}^{3}$	71.43	5.95	8.33		
IIIc	15	158	68.23	5.65	7.87	$C_{20}H_{20}N_2O_4$	68.18	5.68	7.95		
IIId	62	175	56.85	4.46	6.89	$C_{19}H_{17}BrN_2O_3$	56.86	4.24	6.98		
IIIe	16	131	56.89	4.30	6.87	$C_{19}H_{17}BrN_2O_3$	56.86	4.24	6.98		
IIIf	11	124	63.83	4.68	7.77	$C_{19}H_{17}CIN_2O_3$	63.90	4.77	7.80		
IIIg	49	149	62.20	5.71	7.95	$C_{20}H_{20}N_2O_4$	68.18	5.68	7.95		
IIIh	27	137	62.03	5.01	7.28	$C_{20}H_{19}CIN_2O_4$	62.09	4.95	7.24		
IIIi	46	151	52.41	3.70	6.38	$C_{19}H_{16}BrClN_2O_3$	52.35	3.67	6.43		
IVa	8	102	70.76	5.62	8.72	$C_{19}H_{18}N_2O_3$	70.80	5.59	8.69		
VIII	20	181	64.07	4.37	7.72	$C_{19}H_{15}CIN_2O_3$	64.31	4.23	7.89		
IXa	11	86	81.42	6.15	_	$C_{19}H_{18}O_3$	81.63	6.12	_		
IXb	16	94	64.23	4.60	_	$C_{19}H_{17}BrO_3$	64.34	4.55	_		
XIIb	9	119	62.28	3.41	_	$C_{17}H_{11}BrO_2$	62.38	3.36	_		

Table 1. Yields, melting points, and elemental analyses of compounds IIIa-IIIi, IVa, VIII, IXa, IXb, and XIIb

Thus, the reaction of ethyl diazoacetate with 1,3-diphenylpropenone follows the Auwers pattern and regioselectively yields 4,5-dihydro-3*H*-pyrazole (**V**); 1,3-hydride shift in the latter leads to formation of isomeric 4,5-dihydro-1*H*-pyrazole derivatives **IIIa** and IVa at a ratio of 5:1. In the reactions of ester I with 1.3-diarylpropenones **Ib-Ii** we succeeded in isolating only the major products, pyrazoles IIIb-IIIi. Their structure was established on the basis of their elemental compositions (Table 1) and spectral parameters (Table 2). Compounds **IIIb**–**IIIi** showed in the <sup>1</sup>H NMR spectra doublet signals at  $\delta$  4.40–4.48 and 5.13– 5.22 ppm (CDCl<sub>3</sub>), J = 4 Hz, from protons at C<sup>4</sup> and C<sup>5</sup> of the pyrazole ring, which are arranged trans with respect to each other. The <sup>1</sup>H NMR spectra of the reaction mixtures obtained from ethyl diazoacetate and 1,3-diarylpropenones also contained signals from 4-H and 5-H of isomeric pyrazoles **IVb**–**IVi** at δ 4.40 and 5.00 ppm (J = 4 Hz). The signal positions and isomer ratios III: IV are given in Experimental. In

the reaction of ethyl diazoacetate with 3-(2-chlorophenyl)-1-phenylpropenone (**IIf**), apart from products **IIIf** and **IVf**, 20% of ethyl 3-benzoyl-4-(2-chlorophenyl)pyrazole-5-carboxylate (**VIII**) was isolated.

It is known that thermolysis of dihydropyrazoles is a method for preparation of cyclopropane derivatives [6]. According to the data of [3, 7], thermolysis of compound **IIIa** leads to formation of substituted 2-pyranone **XIIa** and ethyl 2-benzoyl-3-phenylcyclopropanecarboxylate whose steric structure was not established. By heating of dihydropyrazoles **IIIa** and **IIId** for 75 min at 220°C, followed by separation of the reaction mixture by column chromatography, we isolated cyclopropanecarboxylates **IXa**, **IXb**, and **Xa** and 2-pyranones **XIIa** and **XIIb** (Scheme 2).

The <sup>1</sup>H NMR spectrum of **IXa** contained three signals from the cyclopropane ring ptotons at  $\delta$  2.65 ( $J_1 = 6$ ,  $J_2 = 10$  Hz), 3.11 ( $J_1 = 6$ ,  $J_2 = 10$  Hz), and 3.39 ppm ( $J_1 = 6$ ,  $J_2 = 7$  Hz). Compound **Xa** showed in the spectrum signals at  $\delta$  2.89 ( $J_1 = 5$ ,  $J_2 = 10$  Hz),

#### Scheme 2.

R = H (a), 3-Br (b).

Table 2	2.	IR	and	$^{1}H$	<b>NMR</b>	spectra	of	compounds	IIIa-IIIi,	IVa.	VI.	VIII.	IXa.	IXb.	Xa.	and	XIIb
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Comp.	IR spectrum, ν, cm <sup>-1</sup>	$^{1}$ H NMR spectrum, $^{a}$ $\delta$ , ppm ( $J$ , Hz)
IIIa <sup>b</sup>	1130, 1250, 1350, 1580, 1690, 1710, 3040, 3370	1.25 t (3H, 7), 4.11 q (2H, 7), 4.49 d (1H, 4), 5.22 d (1H, 4), 7.03 s (1H), 7.31–7.72 (8H), 7.92 d (2H, 7)
IIIb	1130, 1250, 1350, 1580, 1690, 1710, 3040, 3370	1.09 t (3H, 7), 4.02 q (2H, 7), 4.31 d (1H, 4), 5.42 d (1H, 4), 7.07 d (2H, 8), 7.18 d (2H, 8), 7.55 m (2H), 7.70 m (1H), 7.90 m (2H), 8.91 s (1H)
IIIc	1120, 1190, 1260, 1330 1600, 1690, 1710, 3040, 3370	1.10 t (3H, 7), 3.72 s (3H), 4.00 q (2H, 7), 4.30 d (1H, 4), 5.40 d (1H, 4), 6.93 d (2H, 8), 7.11 d (2H, 8), 7.53–7.70 (3H), 7.90 m (2H), 8.83 s (1H)
IIId	1130, 1250, 1350, 1580, 1690, 1710, 3040, 337	1.08 t (3H, 7), 4.04 q (2H, 7), 4.44 d (1H, 4), 5.58 d (1H, 4), 7.12–7.78 (7H), 7.90 d (2H, 7), 9.07 s (1H)
IIIe	1130, 1250, 1350, 1580, 1690, 1710, 3040, 3370	1.20 t (3H, 7), 4.11 q (2H, 7), 4.45 d (1H, 4), 5.18 (1H, 4), 7.03 s (1H), 7.20–7.71 (7H), 7.80 d (2H, 7)
IIIf	1130, 1260, 1350, 1590, 1690, 1710, 3040, 3370	1.24 t (3H, 7), 4.20 q (2H, 7), 4.45 d (1H, 4), 5.10 d (1H, 4), 7.03–7.60 (7H), 8.00 d (2H, 7), 9.68 s (1H)
IIIg	1120, 1190, 1260, 1330, 1600, 1690, 1710, 3040, 3370	1.08 t (3H, 7), 3.86 s (3H), 4.00 q (2H, 7), 4.35 d (1H, 4), 5.42 d (1H, 4), 7.08 d (2H, 7), 7.30–7.45 (3H), 7.87 d (2H, 8), 8.91 s (1H)
IIIh	1100, 1190, 1260, 1340, 1590, 1690, 1710, 3040, 3370	1.24 t (3H, 7), 3.82 s (1H), 4.16 q (2H, 7), 4.40 d (1H, 4), 5.13 d (1H, 4), 6.93 d (2H, 8), 6.95 s (1H), 7.28 d (2H, 8), 7.49 d (2H, 8), 7.82 d (2H, 8)
IIIi	1130, 1250, 1350, 1580, 1690, 1710, 3040, 3370	1.21 t (3H, 7), 4.18 q (2H, 7), 4.41 d (1H, 4), 5.18 d (1H, 4), 7.00 s (1H), 7.20–7.60 (6H), 7.81 d (2H, 7)
IVa <sup>c</sup>	1110, 1280, 1410, 1650, 1735, 3050, 3380	1.35 t (3H, 7), 4.28 q (2H, 7), 4.40 d (1H, 4), 4.95 d (1H, 4), 7.01 s (1H), 7.24–7.59 (8H), 8.13 d (2H, 7)
VI	1110, 1280, 1410, 1650, 1735, 3050, 3380	1.35 t (3H, 7), 4.28 q (2H, 7), 4.40 d (0.1H, 4), 4.95 s (1H), 7.01 s (1H), 7.30–7.60 (8H), 8.13 d (2H, 7)
VIII	3040, 3370	1.12 t (3H, 7), 4.18 q (2H, 7), 7.30–8.10 (9H), 14.80 br.s (1H)
IXa	1220, 1310, 1450, 1730	1.14 t (3H, 7), 2.66 d.d (1H, 10, 6), 3.11 d.d (1H, 10, 6), 3.39 d.d (1H, 7, 6), 4.10 q (2H, 7), 7.18–7.65 (8H), 8.06 d (2H, 7)
IXb	1220, 1310, 1450, 1680, 1730	4.10 q (2H, 7), 7.10–7.70 (7H), 8.07 d (2H, 7)
Xa	1540 1620 1720 2040	1.08 t (3H, 7), 2.89 d.d (1H, 10, 5), 3.29 d.d (1H, 10, 6), 3.87 d.d (1H, 6, 5), 4.02 q (2H, 7), 7.10–7.70 (8H), 8.16 d (2H, 7)
XIIb	1540, 1630, 1720, 3040	6.47 s (1H), 6.92 s (1H), 7.34–7.75 (9H)

<sup>&</sup>lt;sup>a</sup> The <sup>1</sup>H NMR spectra of compounds **IIIb**, **IIIc**, **IIId**, **IIIIf**, and **IIIg** were recorded in DMSO-d<sub>6</sub>, and of the others, in CDCl<sub>3</sub>.

3.29 ( $J_1 = 6$ ,  $J_2 = 10$  Hz), and 3.87 ppm ( $J_1 = 5$ ,  $J_2 = 6$  Hz). The spectral data coincide with those reported for the corresponding methyl esters [8]. In the <sup>1</sup>H NMR spectrum of the product mixture obtained by thermolysis of compound **IIIa** we also observed CH signals from one more isomeric cyclopropane ester **XI**,  $\delta$ , ppm (J, Hz): 3.23 (5, 6), 3.35 (6, 10), 3.57 (5, 10); by analogy with the spectrum of methyl t-2-benzoyl-t-3-phenylcyclopropanecarboxylate [9], product **XI** was assigned the t-rans, t-rans-configuration. Analogous cyclopropane proton signals were also

observed in the <sup>1</sup>H NMR spectrum of the thermolysis products of dihydropyrazole **IIId**. In both cases the ratio **IX**:**X**:**XI**:**XII** was 2:1:1:1. Thus the thermolysis of compounds **III** is not stereospecific.

## **EXPERIMENTAL**

The IR spectra were recorded on a UR-20 spectrometer from 2% solutions in chloroform. The <sup>1</sup>H NMR spectra were obtained on a Bruker DPX-300 instrument (300 MHz) from 2% solutions in CDCl<sub>3</sub>

<sup>&</sup>lt;sup>b</sup> <sup>13</sup>C NMR spectrum,  $\delta_C$ , ppm: 14.0 (CH<sub>3</sub>), 54.7 (CH), 61.2 (CH<sub>2</sub>), 73.5 (CH), 127.5, 128.0, 128.9, 129.0, 129.3, 132.7, 134.1, 139.1, 161.2 (C=O), 195.6 (C=O); UV spectrum,  $\lambda_{max}$ , nm (log ε): 248 (4.17), 290 (4.01).

<sup>&</sup>lt;sup>c</sup> <sup>13</sup>C NMR spectrum,  $\delta_C$ , ppm: 13.9 (CH<sub>3</sub>), 53.4 (CH), 62.1 (CH<sub>2</sub>), 69.4 (CH), 127.0, 127.4, 127.9, 128.8, 129.2, 136.7, 139.4, 171.1 (C=O), 186.4 (C=O); UV spectrum,  $\lambda_{max}$ , nm (log ε): 260 (3.89), 326 (4.08).

or DMSO- $d_6$ . The UV spectra were measured on a Specord UV-Vis spectrophotometer.

Ethyl 5-benzoyl-4-phenyl-4,5-dihydro-1*H*-pyrazole-3-carboxylate (IIIa). 1,3-Diphenylpropenone (IIa), 10 g (0.05 mol), was dissolved in 30 ml of heptane, and 10.9 g (0.1 mol) of ethyl diazoacetate (I) was added. The mixture was heated for 7 h at 75°C, and the solvent was distilled off. By fractional crystallization of the residue from alcohol we isolated 5.75 g (36%) of compound IIIa and 1.2 g (8%) of ethyl 3-benzoyl-4-phenyl-4,5-dihydro-1*H*-pyrazole-5-carboxylate (IVa).

Ethyl 5-benzoyl-4-(4-methylphenyl)-4,5-dihydro-1*H*-pyrazole-3-carboxylate (IIIb) was synthesized in a similar way from 4.44 g (0.02 mol) of 3-(4-methylphenyl)-1-phenylpropenone and 4.56 g (0.04 mol) of ester I. Yield 3.2 g.

Ethyl 5-benzoyl-4-(4-methoxyphenyl)-4,5-dihydro-1*H*-pyrazole-3-carboxylate (IIIc) was synthesized in a similar way from 1 g (4 mmol) of 3-(4-methoxyphenyl)-1-phenylpropenone (IIc) in 10 ml of toluene and 0.95 g (8.4 mmol) of ester I. Recrystallization of the product gave 0.22 g of compound IIIc.

Pyrazoles **IIIe**–**IIIi** were obtained in a similar way.

Ethyl 5-deutero-3-benzoyl-4-phenyl-4,5-dihydro-1*H*-pyrazole-5-carboxylate (VI). 1,3-Diphenylpropenone, 10 g (48 mmol), was dissolved in 30 ml of heptane, and 8.5 g (74 mmol) of ethyl diazoacetate-*d* was added. The mixture was heated for 7 h at 75°C and evaporated. By fractional crystallization from alcohol we isolated 0.55 g (4%) of pyrazole VI, mp 103°C, and 3.5 g (22%) of compound IIIa.

**Ethyl 3-benzoyl-4-phenyl-1***H***-pyrazole-5-car-boxylate (VII).** Compound **IVa**, 1 g (31 mmol), was dissolved in 30 ml of acetic acid, and 1 ml of bromine was added. After 45 min, the mixture was poured into 100 ml of water, and the precipitate was filtered off and recrystallized from ethanol. Yield 0.5 g (50%), mp 98°C [4]. Following the same procedure, from 1 g (3.1 mmol) of a mixture of dihydropyrazoles **IVa** and **Va**, 0.76 g (76%) of compound **VII** was obtained.

Thermolysis of ethyl 5-benzoyl-4-phenyl-4,5-di-hydro-1*H*-pyrazole-3-carboxylate (IIIa). Compound IIIa, 2 g, was heated for 75 min at 210–220°C. The mixture was cooled, and the product was recrystallized from alcohol. Yield of ethyl *c*-2-benzoyl-*t*-3-phenylcyclopropanecarboxylate (IXa) 0.2 g (11%), mp 86°C. The residue was subjected to column chromatography on Al<sub>2</sub>O<sub>3</sub> using hexane–ether (1:1,

by volume) as eluent to isolate 20 mg of ethyl *t*-2-ben-zoyl-*c*-3-phenylcyclopropanecarboxylate (**Xa**).

Thermolysis of ethyl 5-benzoyl-4-(3-bromophenyl)-4,5-dihydro-1*H*-pyrazole (IIId). Compound IIId, 2 g, was heated for 75 min at 210–220°C. After cooling, the resulting material was subjected to column chromatography on Al<sub>2</sub>O<sub>3</sub> using hexane–ether, (1:1, by volume) as eluent to isolate 0.3 g (16%) of ethyl *c*-2-benzoyl-*t*-3-(3-bromophenyl)cyclopropane-carboxylate (IXb), mp 94°C, and 0.15 g (9%) of 4-(3-bromophenyl)-6-phenyl-2*H*-2-pyranone (XIIb), mp 119°C.

The ratio of isomeric dihydropyrazoles **III** and **IV** was determined from the <sup>1</sup>H NMR spectra of the reaction mixtures. Below are given compound no., chemical shifts of the 4-H and 5-H protons (δ, ppm), and **III**:**IV** isomer ratios: **IIIa**, 5.22, 4.48, **IVa**, -, 4.91, 5; **IIIb**, 5.19, 4.46, **IVb**, 4.38, 4.88, 4; **IIIc**, 5.17, 4.44, **IVc**, 4.36, 4.87, 2; **IIId**, 5.21, 4.46, **IVd**, 4.38, 4.88, 5; **IIIe**, 5.19, 4.47, **IVe**, 4.38, 4.89, 3; **IIIf**, 5.49, 5.02, **IVf**, 4.46, 5.12, 3; **IIIg**, 5.18, 4.49, **IVg**, 4.38, 4.94, 4; **IIIh**, 5.14, 4.40, **IVh**, 4.38, 4.86, 5; **IIIi**, 5.18, 4.42, **IVi**, -, 4.85, 2.

#### REFERENCES

- Molchanov, A.P., Stepakov, A.V., and Kostikov, R.R., Russ. J. Org. Chem., 2000, vol. 36, no. 8, pp. 1139– 1143
- 2. Levai, A., *Khim. Geterotsikl. Soedin.*, 1997, no. 6, pp. 747–759.
- 3. Kohler, E.P. and Steele, L.L., *J. Am. Chem. Soc.*, 1919, vol. 41, no. 7, pp. 1093–1108.
- 4. El Sayed El Kholy, I., Mishrikey, M.M., Fuid-Alla, H.M., and Nashed, M.A., *J. Heterocycl. Chem.*, 1979, vol. 16, no. 5, pp. 849–853.
- 5. Domnin, I.N. and Kostikov, R.R., USSR Inventor's Certificate no. 560 879, 1977; *Byull. Izobret.*, 1977, no. 21.
- 6. Buschby, R.J., *Methoden der organischen Chemie* (*Houben–Weyl*), de Meijere, A., Ed., Stuttgart: Georg Thieme, 1997, 4th ed., vol. E17b, p. 1059.
- El Sayed El Kholy, I., Rafla, F.K., and Mishrikey, M.M., *J. Chem. Soc. C*, 1970, no. 11, pp. 1578– 1584.
- Adams, J., J. Org. Chem., 1970, vol. 35, pp. 1600– 1604.
- 9. Alcaide, B., Casarrubios, L., Dominguez, G., Retamosa, A., and Sierra, M., *Tetrahedron*, 1996, vol. 52, no. 41, pp. 13215–13226.